# CDA 3200 Digital Systems 

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## Outline

- Minimum Forms of Switching Functions
- Two- and Three-Variable Karnaugh Maps
- Four-Variable Karnaugh Maps


## Minimum Forms of Switching Functions (1/5)

- A minimum sum-of-products expression for a function is designed as a sum of product terms which
- Has a minimum number of terms
- Has a minimum number of literals


## Minimum Forms of Switching Functions (2/5)

- The logic algebraic techniques can be used to simplify a logic expression to its minimum sum-of-products.
- However, the procedures are difficult to apply in a systematic way and it is difficult to tell when you have arrived at a minimum solution.


## Minimum Forms of Switching Functions (3/5)

- Given a minterm expansion, the minimum sum-of-products form can often be obtained by the following procedure:
- Combine terms by using $X Y^{\prime}+X Y=X$ to eliminate as many terms as possible.
- Eliminate redundant terms by using the consensus theorem or other theorems.
- The result may depend on the order in which terms are combined or eliminated.


## Minimum Forms of Switching Functions (4/5)

- Example:
- $F(a, b, c)=s u m[m(0,1,2,5,6,7)]$
$-=a^{\prime} b^{\prime} c^{\prime}+a^{\prime} b^{\prime} c+a^{\prime} b c^{\prime}+a b{ }^{\prime} c+a b c$ '+abc
$-=a^{\prime} b^{\prime} c^{\prime}+\underline{a^{\prime}} b^{\prime} c+\underline{a}{ }^{\prime} b^{\prime} c+a^{\prime} b c^{\prime}+a b^{\prime} c+a b c^{\prime}+a b c^{\prime}+a b c$
$-=a^{\prime} b^{\prime}+b^{\prime} c+b c^{\prime}+a b$


## Minimum Forms of Switching Functions (5/5)

- Example: (cont)
- $F(a, b, c)=s u m[m(0,1,2,5,6,7)]$
$-=a^{\prime} b^{\prime} c^{\prime}+a^{\prime} b^{\prime} c+a a^{\prime} b c^{\prime}+a b^{\prime} c+a b c^{\prime}+a b c$
$-=a^{\prime} b^{\prime}+b c^{\prime}+a c$


## Two- and Three-Variable Karnaugh Maps (1/10)

- In a Karnaugh map, minterms in adjacent squares of the map can be combined since they differ in only one variable. The combinable terms are looped in the Karnaugh map.

| $A B$ | $F$ |
| :--- | :--- |
| 00 | 1 |
| 01 | 1 |
| 10 | 0 |
| 11 | 0 |


| $B^{A} 0$ |  |  |
| :---: | :---: | :---: |
| 0 | 1 | 0 |
| 1 | 1 | 0 |
| (b) |  |  |

## Two- and Three-Variable Karnaugh Maps (2/10)

- In a three-variable (A,B,C) Karnaugh map, the value of one variable $A$ is listed across the top of the map, and the values of the other two variables ( $B, C$ ) are listed along the side of the map.
- Note the rows are labeled in the sequence $00,01,11,10$, why?


## Two- and Three-Variable Karnaugh Maps (3/10)

| $A$ | $B$ | $C$ | $F$ |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |


$A B C=001, F=0$

## Two- and Three-Variable Karnaugh Maps (4/10)

- In a three-variable Karnaugh map, the top and bottom rows of the map are defined to be adjacent because the corresponding minterms in these rows differ in only one variable.


## Two- and Three-Variable Karnaugh

 Maps (5/10)| $b c$ | 0 | 1 |  |
| :---: | :---: | :---: | :---: |
| 00 | 000 |  | 100 is adjacent to 110 |
| 01 | 001 | 101 |  |
| 11 | 011 | >111 |  |
|  | 4 |  |  |
|  | 10 |  |  |
| 10 | 010 | 110 |  |

(a) Binary notation

(b) Decimal notation

## Two- and Three-Variable Karnaugh Maps (6/10)

- How would you loop minterms in this
Karnaugh map?



## Two- and Three-Variable Karnaugh Maps (7/10)

- How to plot 1s in a Karnaugh map for the following expressions and loop all you can loop:
$-F(a, b, c)=a^{\prime} b c+a b c^{\prime}+a b c+a^{\prime} b c^{\prime}$
$-F(a, b, c)=a b c^{\prime}+b^{\prime} c+a^{\prime}$
$-F(a, b, c)=b^{\prime} c^{\prime}+a b+b c{ }^{\prime}$
$-F(a, b, c)=a b+a^{\prime} c$


## Two- and Three-Variable Karnaugh Maps (8/10)

- Two terms in adjacent squares on the map differ in only one variable and can be combined using the theorem $X Y^{\prime}+\mathrm{XY}=\mathrm{X}$
- Two adjacent "loops" that differ only one variable can be combined.


## Two- and Three-Variable Karnaugh Maps (9/10)

-The Karnaugh map can also illustrate the consensus theorem $X Y+X^{\prime} Z+Y Z=X Y+X^{\prime} Z$



$$
x y+x^{\prime} z+y z=x y+x^{\prime} z
$$

## Two- and Three-Variable Karnaugh Maps (10/10)

- The simplification using Karnaugh maps can also result in different solutions.



## Four-Variable Karnaugh Maps (1/9)

- Are $\mathrm{m}_{8}$ and $\mathrm{m}_{0}$ adjacent?
- Are $m_{2}$ and $m_{10}$ adjacent?
- Are $l o o p_{0 \& 8}$ and loop $_{2 \& 10}$ adjacent?



## Four-Variable Karnaugh Maps (2/9)

- Minterms can be combined in group of 2, 4 , or 8 to eliminate 1,2 , or 3 variables.

(b)


## Four-Variable Karnaugh Maps (3/9)

|  |  | 00 | 01 | 11 |
| :--- | :--- | :--- | :--- | :--- |
| ${ }^{\infty}$ | 10 | 10 | 1 | 1 |
| ${ }^{\circ}$ | 1 | 1 | 1 |  |
| ${ }^{\circ}$ | 1 | 1 | 1 |  |
| 11 |  |  |  |  |
| ${ }^{10}$ |  |  |  |  |

Anything wrong?

## Four-Variable Karnaugh Maps (4/9)

|  | 0 | 1 |
| :---: | :---: | :---: |
| $\infty$ | 1 |  |
| ${ }^{1}$ | 1 |  |
| ${ }^{11}$ | 1 |  |
| ${ }^{10}$ |  |  |

Anything wrong?

## Four-Variable Karnaugh Maps (5/9)

- Minterms can be combined in group of 2, 4 , or 8 to eliminate 1, 2, or 3 variables.
- The number of minterms, contained in a loop, can only be a power of 2 .


## Four-Variable Karnaugh Maps (6/9)

- $F(a, b, c, d)=a^{\prime} b+a c d+d^{\prime}$


Can you simplify this further?

## Four-Variable Karnaugh Maps (7/9)

- Minterm Expansion: $F(a, b, c, d)=b c^{\prime}+a^{\prime} b^{\prime} d+a b ' c d^{\prime}$


| $C D$ | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | 4 | 12 | 8 |
| 01 | 1 | 5 | 13 | 9 |
| 11 | 3 | 7 | 15 | 11 |
| 10 | 2 | 6 | 14 | 10 |

## Four-Variable Karnaugh Maps (8/9)

- Extension to functions with "don't care" terms
- "do not care" terms are indicated by X's in Karnaugh map.
- The X's are only used if they will simpify the resulting expression.


## Four-Variable Karnaugh Maps (9/9)



