Truth Tables (again)

Recall that a boolean equation can be represented by a Truth Table

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

A truth table for a boolean function of N variables has 2^N entries.

The '1's represent F(A,B,C).

The '0's represent F'(A,B,C)

Truth Table to SOP Form

Can write SOP form of equation directly from truth table.

A	B	C	F	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	← A'BC
1	0	0	1	← AB'C'
1	0	1	1	← AB'C
1	1	0	1	← ABC'
1	1	1	1	← ABC

F(A,B,C) = A'BC + AB'C' + AB'C + ABC' + ABC

Note that each term in has ALL variables present. If a product term has ALL variables present, it is a MINTERM.

Truth Table to POS Form

To get POS form of F, write SOP form of F', then use DeMorgan's Law.

A	B	C	F	
0	0	0	0	← A'B'C'
0	0	1	0	← A'B'C
0	1	0	0	← A'BC'
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	1	

F'(A,B,C) = A'B'C' + A'B'C + A'BC'

Take complement of both sides:

(F'(A,B,C))' = (A'B'C' + A'B'C + A'BC')'

Apply DeMorgan's Law to right side.
Left side is (F')' = F.

F(A,B,C) = (A'B'C')' (A'B'C)' (A'BC')'

apply DeMorgan's Law to each term

F(A,B,C) = (A+B+C) (A+B+C')(A+B'+C)

POS Form!!

Minterms, Maxterms

We saw that:

F(A,B,C) = A'BC + AB'C' + AB'C + ABC' + ABC' + ABC

SOP form. If a product term has all variables present, it is a MINTERM.

F(A,B,C) = (A+B+C) (A+B+C')(A+B'+C)

POS form. If a sum term has all variables present, it is a MAXTERM.

All Boolean functions can be written in terms of either Minterms or Maxterms.

Minterm, Maxterm Notation

Each line in a truth table represents both a Minterm and a Maxterm.

Row No.	A B C	Minterms	Maxterms
0	0 0 0	A'B'C' = m_0	A+B+C = M_0
1	0 0 1	A'B'C = m_1	A+B+C' = M_1
2	0 1 0	A'B C' = m_2	A+B'+C = M_2
3	0 1 1	A'B C = m_3	A+B'+C' = M_3
4	1 0 0	A B'C' = m_4	A'+B+C = M_4
5	1 0 1	A B'C = m_5	A'+B+C'= M_5
6	1 1 0	A B C' = m_6	A'+B'+C = M_6
7	1 1 1	A B C = m_7	A'+B'+C' = M_7

Using Minterms, Maxterms

A boolean function can be written in terms of Minterm or Maxterm notation as a shorthand method of specifying the function.

F(A,B,C) = A'BC + AB'C' + AB'C + ABC' + ABC' + ABC
$= m_3 + m_4 + m_5 + m_6 + m_7$
$= \Sigma\, m(3,4,5,6,7)$

F(A,B,C) = (A+B+C) (A+B+C')(A+B'+C)
$= M_0\, M_1\, M_2$
$= \prod M(0,1,2)$

Minterms correspond to '1's of F, Maxterms correspond to '0's of F in truth table.

From Minterms to Truth Table

Minterms correspond to '1's in Truth table

F(A,B,C) = Σ m(1,2,6)
$= m_1 + m_2 + m_6$
= A'B'C + A'BC' + ABC'

	A	B	C	F
	0	0	0	0
m_1 →	0	0	1	1
m_2 →	0	1	0	1
	0	1	1	0
	1	0	0	0
	1	0	1	0
m_6 →	1	1	0	1
	1	1	1	0

From Minterms to Maxterms to Truthtable

To go from Minterms to Maxterms, list the numbers that are NOT present (with 3 variables, minterm/maxterm numbers range from 0 to 7

F(A,B,C) = Σ m(1,2,6)
= $\prod$ M(0,3,4,5,7)
= (A+B+C)(A+B'+C')(A'+B+C)
(A'+B+C')(A'+B'+C')

	A	B	C	F
M_0 →	0	0	0	0
	0	0	1	1
	0	1	0	1
M_3 →	0	1	1	0
M_4 →	1	0	0	0
M_5 →	1	0	1	0
	1	1	0	1
M_7 →	1	1	1	0

Maxterms correspond to '0's in Truth table

Examples

F(A,B,C,D) = Σ m(0) (minterm form)
= A'B'C'D' (SOP form)

= $\prod$ M(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)
(maxterm form)

(POS form too long to write…..)

F(A,B) = Σ m(1,2) (minterm form)
= A'B + AB' (SOP form)
= $\prod$ M(0,3) (maxterm form)
= (A+B)(A'+B') (POS form)
= A xor B (did you recognize this?)

Minterm Expansion

A *minterm* must have every variable present. If a boolean product term does not have every variable present, then it can be *expanded* to its minterm representation.

F(A,B,C) = AB + C   neither AB, or C are minterms

To expand AB to minterms, use the relation:

AB = AB(C+ C') = ABC + ABC'

To expand C to minterms, do:

C = C(A+A') = AC+ A'C = AC(B+B') + A'C(B+B')

= ABC + AB'C + A'BC + A'B'C

F = AB +C = A'B'C + A'BC + AB'C + ABC' + ABC

F(A,B,C) = Σ m(1,3,5,6,7)

Maxterm Expansion

A *maxterm* must have every variable present. If a boolean sum term does not have every variable present, then it can be *expanded* to its *maxterm* representation.

F(A,B,C) = (A+B) (C)   neither (A+B), or C are maxterms

To expand (A+B) to maxterms, use the relation:

(A+B) = (A+B+C'C) = (A+B+C')(A+B+C)

To expand C to minterms, do:

C = C+A'A = (A'+C)(A+C) = (A'+BB'+C)(A+C+BB')

= (A'+B'+C)(A'+B+C)(A+B'+C)(A+B+C)

F = (A+B)(C) = (A+B+C)(A+B+C')(A+B'+C)(A'+B+C)(A'+B'+C)

F(A,B,C) = Π M(0,1,2,4,6)

Minimize from Minterm From

Y = Σ m(3,4,5,6,7)

Y = A'BC + AB'C' + AB'C + ABC' + ABC

Look for differences in only one variable

Y = A'BC + AB'(C' + C) + AB(C' + C)

= A'BC + AB' + AB

= A'BC + A(B' +B)

= A'BC + A

= BC + A

A difference in only one variable is called a Boolean Adjacency.

Minimize from POS

$Y = \Pi\ M(0,1,2)$
$Y = (A+B+C)(A+B+C')(A+B'+C)$

Again, look for differences in only one variable

$Y = (A+B + CC')\ (A+B+C)$
$= (A+B)(A+B'+C)$
$= (A+B)((A+C) + B')$
$= (A+B)(A+C) + (A+B)B'$
$= A + BC + AB + BB'$
$= A + AB + BC$
$= A(1 + B) + BC$
$= A + BC$

Karnaugh Maps

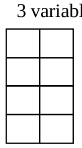
- Karnaugh Maps (K-Maps) are a graphical method of visualizing the 0's and 1's of a boolean function
  - K-Maps are very useful for performing Boolean minimization.
- Will work on 2, 3, and 4 variable K-Maps in this class.
- Karnaugh maps can be easier to use than boolean equation minimization once you get used to it.

K-Maps

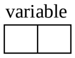
A K-map has a square for each '1' or '0' of a boolean function.

One variable K-map has $2^1 = 2$ squares.
Two variable K-map has $2^2 = 4$ squares
Three variable K-map has $2^3 = 8$ squares
Four variable K-map has $2^4 = 16$ squares

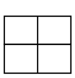
3 variable



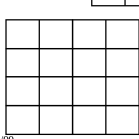
1 variable



2 variable



4 variable



Plotting Functions on K-Maps

Each square represents a row in the truth table. The values in each square is the value of F from the truth table.

Row	A	F(A)
0	0	?
1	1	?

A=0	A=1
?	?

A=0	A=1
r0	r1

Row	A	F(A)
0	0	0
1	1	1

A=0	A=1
0	1

F(A) = A

Row	A	F(A)
0	0	1
1	1	0

A=0	A=1
1	0

F(A) = A'

Plotting 2-Variable Functions

Row	A B	F(A,B)
0	0 0	?
1	0 1	?
2	1 0	?
3	1 1	?

B \ A	0	1
0	?	?
1	?	?

B \ A	0	1
0	r0	r2
1	r1	r3

Row 0 from TT, A=0, B= 0

Row	A B	F(A,B)
0	0 0	0
1	0 1	1
2	1 0	1
3	1 1	1

B \ A	0	1
0	0	1
1	1	1

F(A,B) = A + B

Plotting 2-Variable Functions (cont.)

Row	A B	F(A,B)
0	0 0	0
1	0 1	1
2	1 0	1
3	1 1	0

B \ A	0	1
0	0	1
1	1	0

F(A,B) = A'B + AB'

Row	A B	F(A,B)
0	0 0	0
1	0 1	0
2	1 0	0
3	1 1	1

B \ A	0	1
0	0	0
1	0	1

F(A,B) = AB

Plotting 3-Variable Functions

Row	A B C	F(A,B,C)
0	0 0 0	?
1	0 0 1	?
2	0 1 0	?
3	0 1 1	?
4	1 0 0	?
5	1 0 1	?
6	1 1 0	?
7	1 1 1	?

BC \ A	0	1
00	?	?
01	?	?
11	?	?
10	?	?

BC \ A	0	1
00	r0	r4
01	r1	r5
11	r3	r7
10	r2	r6

Boolean Adjacency

Note on the three variable map:

BC \ A	0	1
00	r0	r4
01	r1	r5
11	r3	r7
10	r2	r6

Correct

BC \ A	0	1
00	r0	r4
01	r1	r5
10	r2	r6
11	r3	r7

WRONG!!!

Each square on the 3-variable map is Boolean Adjacent. Adjacent squares only differ by ONE BOOLEAN VARIABLE!!!

Boolean Adjacency

BC \ A	0	1
00	f(A'B'C')	f(AB'C')
01	f(A'B'C)	f(AB'C)
11	f(A'BC)	f(ABC)
10	f(A'BC')	f(ABC')

Squares at bottom of map adjacent to squares top of map.

Each square is boolean adjacent to neighbor.

Plotting 3-Variable Functions

Row	A B C	F(A,B,C)
0	0 0 0	1
1	0 0 1	0
2	0 1 0	1
3	0 1 1	0
4	1 0 0	0
5	1 0 1	0
6	1 1 0	1
7	1 1 1	0

BC \ A	0	1
00	1	0
01	0	0
11	0	0
10	1	1

F(A,B,C) = Σ m(0,2,6)

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Another 3-variable Example

Row	A B C	F(A,B,C)
0	0 0 0	0
1	0 0 1	0
2	0 1 0	0
3	0 1 1	0
4	1 0 0	1
5	1 0 1	1
6	1 1 0	1
7	1 1 1	0

BC \ A	0	1
00	0	1
01	0	1
11	0	0
10	0	1

F(A,B,C) = Σ m(4,5,6)

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Plotting 4-Variable Functions

Row	A B C D	F(A,B,C,D)
0	0 0 0 0	?
1	0 0 0 1	?
2	0 0 1 0	?
3	0 0 1 1	?
4	0 1 0 0	?
5	0 1 0 1	?
6	0 1 1 0	?
7	0 1 1 1	?
8	1 0 0 0	?
9	1 0 0 1	?
10	1 0 1 0	?
11	1 0 1 1	?
12	1 1 0 0	?
13	1 1 0 1	?
14	1 1 1 0	?
15	1 1 1 1	?

CD \ AB	00	01	11	10
00	?	?	?	?
01	?	?	?	?
11	?	?	?	?
10	?	?	?	?

CD \ AB	00	01	11	10
00	r0	r4	r12	r8
01	r1	r5	r13	r9
11	r3	r7	r15	r11
10	r2	r6	r14	r10

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Boolean Adjacency

CD \ AB	00	01	11	10
00	f(A'B'C'D')	f(A'BC'D')	f(ABC'D')	f(AB'C'D')
01	f(A'B'C'D)	f(A'BC'D)	f(ABC'D)	f(AB'C'D)
11	f(A'B'CD)	f(A'BCD)	f(ABCD)	f(AB'CD)
10	f(A'B'CD')	f(A'BCD')	f(ABCD')	f(AB'CD')

Squares at bottom of map adjacent to squares top of map and vice-versa.

Squares at left edge are adjacent to squares at right edge and vice-versa.

Plotting 4-Variable Functions

Row	A B C D	F(A,B,C,D)
0	0 0 0 0	0
1	0 0 0 1	0
2	0 0 1 0	1
3	0 0 1 1	1
4	0 1 0 0	0
5	0 1 0 1	0
6	0 1 1 0	1
7	0 1 1 1	0
8	1 0 0 0	0
9	1 0 0 1	0
10	1 0 1 0	1
11	1 0 1 1	0
12	1 1 0 0	0
13	1 1 0 1	0
14	1 1 1 0	0
15	1 1 1 1	1

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	0	1	0
10	1	1	0	1

$F = \Sigma\, m(2,3,6,10,15)$

What do you need to Know?

- Minterm, Maxterm definitions
- Truth table to Minterms, vice versa
- Truth table to Maxterms, vice versa
- Minterms to Maxterms, vice versa
- Plotting 2,3,4 variable functions on K-Maps